International Journal of Engineering Research and Reviews ISSN 2348-697X (Online)
Vol. 11, Issue 1, pp: (20-26), Month: January - March 2023, Available at: www.researchpublish.com

Mathematical Modelling for Pulping System in Paper industry

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DOI: https://doi.org/10.5281/zenodo.7639148

Published Date: 14-February-2023

Abstract: **The paper discusses the pulping system in paper industry consisting of four subsystems in series. Failure and repair rates in various subsystems are taken to be constant. Formulation of the problem is carried out using simple probability considerations. Tables and graphs for various parameters are given which are useful to management for maintenance planning.**

Keywords: **Mathematical Modelling, pulping system, paper industry.**

1. INTRODUCTION

A system is kept failure free (as far as possible) under the given operative conditions to achieve the goal of production and long run availability. A probabilistic analysis of the system under given operative conditions is helpful for design and maintenance purposes for optimization of the system working. In paper industry, we have various subsystems—chipping, feeding, pulp preparation, washing, screening, bleaching, preparation of paper, collection. The important part of the paper industry is the preparation of pulp which is the subject of our present discussion.

The pulp preparation system needs the operations cooking of chips, separation of knots, washing of pulp by liquor and opening of fibers. The chips from storage are fed into the digester through feeding system discussed in Ref. 1. Mixed with white liquor, cooking of chips is done for several hours in the digester using a team heating system. Since the presence of knots precludes the production of paper, these are removed from the cooked chips by passing the pulp through the knotter. The knot-free pulp is then passed over deckers arranged in series. Vacuum is maintained in the drum of deckers and the pulp rolls on the surface. The purpose of the deckers is to remove the used liquor (called black liquor) from the pulp to the maximum. Washing of the pulp is done in two to three stages. The pulp is then passed through the opener rotating at high speed. Here the fibers are separated through a combing action. The prepared pulp with fine fibers is sent to the screening section.

2. THE MODEL

The pulping system is comprised of four subsystem:

(i) the digester (A) for cooking the chips whose failure causes complete failure of the cooking system,

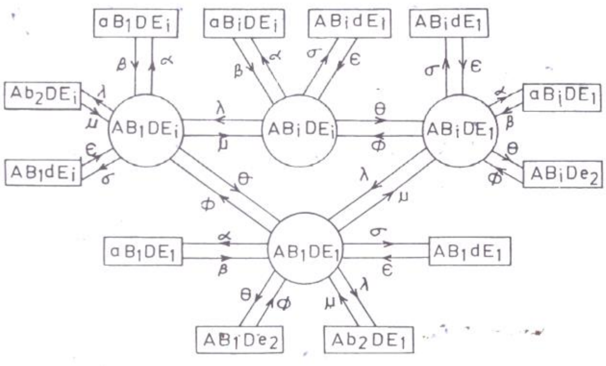

Fig. 1. Transition diagram of the system

(ii) the knotter (B) having one unit in standby whose complete failure occurs only if both the units fail,

(iii) the deckers (D), three in number and arranged in series; failure of any one causes complete failure of the washing process. Although production is possible with two or a single stage decker, this produces low quality paper which is not required due to profit reasons, and

(iv) the opener € having one unit in standby whose complete failure occurs only when both the units fail.

The transition diagram of the system is given in Fig. 1.

3. MATHEMATICAL MODELLING

Modelling is done taking the following assumptions and notations:

(i) Failure and repair rates are constant and statistically independent.

(ii) A repaired unit is as good as new.

(iii) Knotter (B) and opener € have standby units one in each.

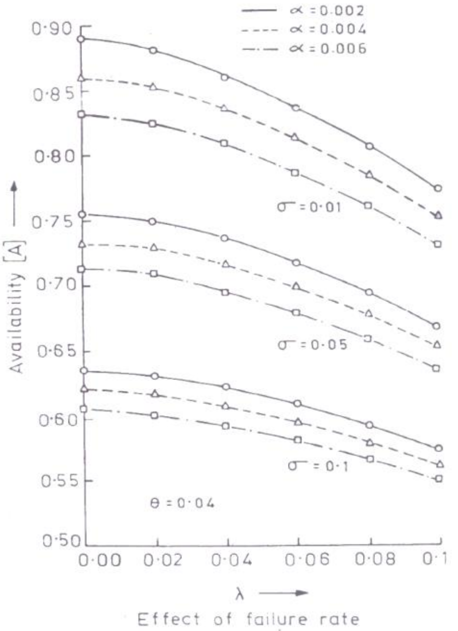

Fig. 2. The effect of failure rate of digester, decker and knotter

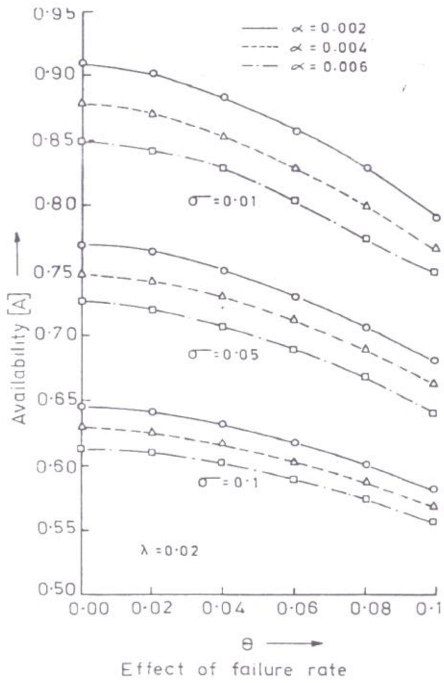

Fig. 3. The effect of failure rate of digester, decker and opener

Table 1: Showing the effect of failure of digester, knotter and decker taking $\theta = 0.04$, $\beta = 0.05$, $\varepsilon = 0.2$, $\phi = 0.2$, $\mu = 0.2$

α	σ	Availability (A)					
		$\lambda = 0.0$	$\lambda = 0.02$	$\lambda = 0.04$	$\lambda = 0.06$	$\lambda = 0.08$	$\lambda = 0.1$
0.002	0.01	0.89021	0.88306	0.86455	0.83853	0.80800	0.77519
0.002	0.05	0.75561	0.75051	0.73710	0.71810	0.69559	0.67114
0.002	0.10	0.63559	0.63194	0.62241	0.60880	0.59255	0.57471
0.004	0.01	0.85960	0.85293	0.83565	0.81132	0.78271	0.75188
0.004	0.05	0.73350	0.72864	0.71599	0.69800	0.67676	0.65359
0.004	0.10	0.61983	0.61636	0.60729	0.59433	0.57883	0.56180
0.006	0.01	0.831.02	0.82479	0.80863	0.78582	0.75894	0.72993
0.006	0.05	0.71259	0.70800	0.69606	0.67909	0.65893	0.63694
0.006	0.10	0.60484	0.60153	0.59289	0.58053	0.56573	0.54945

(iv) The standby units are of the same nature and capacity as the active units.

(v) Each subsystem has separate repair facility. There is no waiting time for repair in the system.

(vi) Repair men are always available, their costs are not considered.

$\alpha, \lambda, \sigma, \theta$ respective constant failure rates of subsystems A, B, D and E.

$\beta, \mu, \varepsilon, \phi$ respective constant repair rates of subsystems A, B, D and E.

A, B, D, E denote good state of the subsystem while the respective small letters denote failed states.

Table 2: Showing the effect of failure of digester, decker and opener taking $\lambda = 0.02$

α	σ	Availability (A)					
		$\lambda = 0.0$	$\lambda = 0.02$	$\lambda = 0.04$	$\lambda = 0.06$	$\lambda = 0.08$	$\lambda = 0.1$
0.002	0.01	0.90984	0.90238	0.88306	0.85592	0.82414	0.79004
0.002	0.05	0.76977	0.76442	0.75051	0.73082	0.70752	0.68224
0.002	0.10	0.64554	0.64177	0.63194	0.61792	0.60119	0.58283
0.004	0.01	0.87789	0.87094	0.85293	0.82759	0.79784	0.76583
0.004	0.05	0.74678	0.74174	0.72863	0.71007	0.68805	0.66412
0.004	0.10	0.62929	0.62571	0.61636	0.60302	0.58707	0.56955
0.006	0.01	0.84811	0.84162	0.82479	0.80108	0.77317	0.74308
0.006	0.05	0.72512	0.72037	0.70800	0.69045	0.66962	0.64197
0.006	0.10	0.61384	0.61043	0.60153	0.58881	0.57360	0.55687

B_1, E_1 denotes that one unit of subsystem B or E is in failed state and the system is working with standby unit.

b_2, e_2 denotes total failure of the system due to failure of second (standby) unit of *B* or *E* while the other unit is in repair.

Probability considerations gives the following governing equations for steady state of the system:

$$\left(\lambda+\theta+\sigma+\alpha\right)P_{AD}^{B_iE_i} = \varepsilon P_{Ad}^{B_iE_i} + \beta P_{aD}^{B_iE_i} + \mu P_{AD}^{B_iE_i} + \phi P_{AD}^{B_iE_i} \quad (1)$$

$$\mu\phi P_{AD}^{B_1E_1} = \lambda\theta P_{AD}^{E_iE_i} \quad (2)$$

$$\mu P_{AD}^{B_iE_i} = \lambda P_{AD}^{B_iE_i} \quad (3)$$

$$\phi P_{AD}^{B_iE_i} = \theta P_{AD}^{B_iE_i} \quad (4)$$

$$\beta P_{aD}^{B_iE_i} = \alpha P_{AD}^{B_1E_i} \quad (5)$$

$$\varepsilon P_{Ad}^{B_iE_i} = \sigma P_{AD}^{B_1E_i} \quad (6)$$

$$\phi P_{AD}^{B_{ie_2}} = \theta P_{AD}^{B_1E_i} \quad (7)$$

$$\varepsilon P_{Ad}^{B_1E_1} = \sigma P_{AD}^{B_1E_1} \quad (8)$$

$$\beta P_{aD}^{B_iE_1} = \alpha P_{AD}^{B_iE_1} \quad (9)$$

$$\varepsilon P_{Ad}^{B_iE_i} = \sigma P_{AD}^{B_iE_i} \quad (10)$$

$$\mu P_{AD}^{b_2E_i} = \lambda P_{AD}^{B_1E_i} \quad (11)$$

$$\beta P_{aD}^{B_1E_1} = \alpha P_{AD}^{B_1E_1} \quad (12)$$

$$\mu P_{AD}^{b_2E_1} = \lambda P_{AD}^{B_1E_1} \tag{13}$$

$$\phi P_{AD}^{B_ie_2} = \theta P_{AD}^{B_iE_1} \tag{14}$$

$$\varepsilon P_{Ad}^{B_iE_1} = \sigma P_{AD}^{B_iE_1} \tag{15}$$

$$\beta P_{aD}^{B_iE_i} = \alpha P_{AD}^{B_iE_i} \tag{16}$$

4. ANALYSIS

Solving the equns (1) to (16) recursively and using normalizing condition, we get

$$P_{AD}^{B_iE_i} = \left[\left(1+\frac{\lambda}{\mu}+\frac{\theta}{\phi}+\frac{\alpha}{\beta}+\frac{\sigma}{\varepsilon}\right)\left(1+\frac{\lambda\theta}{\mu\phi}\right)+\left(\frac{\lambda}{\mu}+\frac{\theta}{\phi}\right)\left(\frac{\alpha}{\beta}+\frac{\sigma}{\varepsilon}\right)+\frac{\lambda^2}{\mu^2}+\frac{\theta^2}{\phi^2}\right]^{-1} = [M]^{-1}$$

Table 3: Showing the effect of failure of knotter and opener for particular case when failure rates in digester and decker are considered □ 0.

λ	Availability (A_1)					
	$\theta = 0.0$	$\theta = 0.02$	$\theta = 0.04$	$\theta = 0.06$	$\theta = 0.08$	$\theta = 0.1$
0.0	1.0000	0.99099	0.96774	0.93525	0.89744	0.85714
0.02	0.99099	0.95395	0.90000	0.84314	0.78864	0.73834
0.04	0.96774	0.90000	0.83333	0.77320	0.72000	0.67308
0.06	0.93525	0.84314	0.77320	0.71561	0.66667	0.62430
0.08	0.89744	0.78864	0.72000	0.66667	0.62232	0.58425
0.10	0.85714	0.73834	0.67308	0.62430	0.58427	0.55000

The steady state availability (A) of the system is given by

$$[A] = P_{AD}^{B_iE_i} + P_{AD}^{B_iE_i} + P_{AD}^{B_1E_i} + P_{AD}^{B_1E_1} = \left[1+\frac{\lambda}{\mu}+\frac{\theta}{\phi}+\frac{\lambda\theta}{\mu\phi}\right][M]^{-1}$$

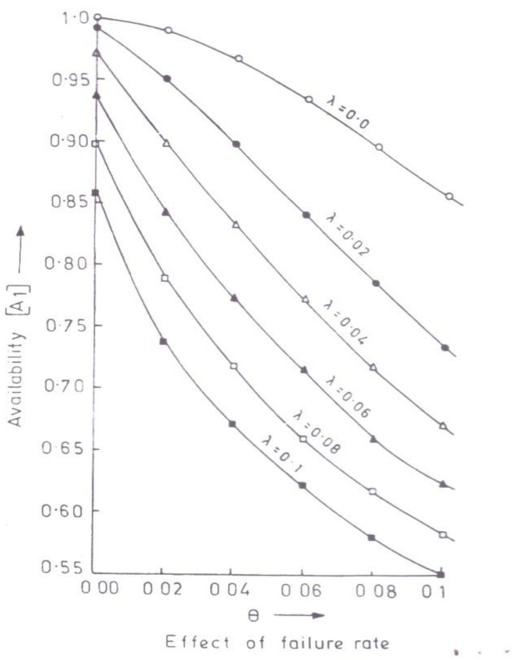

Fig. 4. The effect of failure rates of knotter, and opener

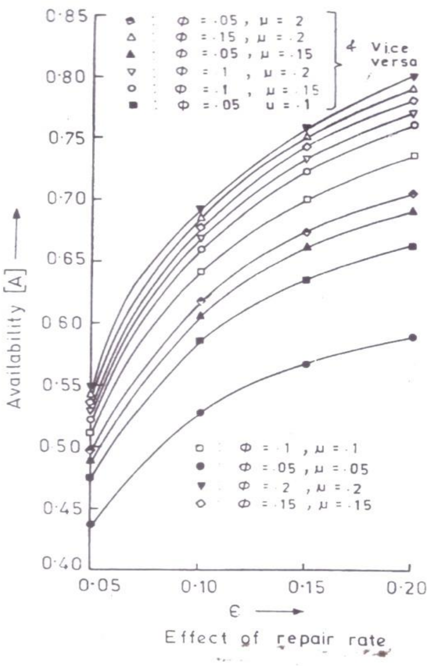

Fig. 3. The effect of failure rates of knotter, decker and opener

Table 4: Showing the effect of repair rates of knotter, decker and opener taking repair rate of digester $\beta = 0.05$, $\alpha = 0.001$, $\lambda = 0.02$, $\theta = 0.02$, $\sigma = 0.04$

ϕ	μ	Availability (A)			
		$\varepsilon = 0.05$	$\varepsilon = 0.10$	$\varepsilon = 0.15$	$\varepsilon = 0.20$
0.05	0.05	0.43657	0.52894	0.56907	0.59151
0.05	0.10	0.47544	0.58708	0.63694	0.66519
0.05	0.15	0.48932	0.60840	0.66211	0.69269
0.05	0.20	0.49634	0.61930	0.67504	0.70685
0.10	0.05	0.47544	0.58708	0.63694	0.66519
0.10	0.10	0.51195	0.64378	0.70423	0.73892
0.10	0.15	0.52310	0.66152	0.72551	0.76239
0.10	0.20	0.52817	0.66964	0.73529	0.77320
0.15	0.05	0.48932	0.60840	0.66211	0.69269
0.15	0.10	0.52310	0.66152	0.72551	0.76239
0.15	0.15	0.53264	0.67685	0.74399	0.78282
0.15	0.20	0.53674	0.68349	0.75202	0.79171
0.20	0.05	0.49634	0.61930	0.67504	0.70685
0.20	0.10	0.52817	0.66964	0.73530	0.77320
0.20	0.15	0.53674	0.68349	0.75202	0.79171
0.20	0.20	0.54032	.68929	0.75910	0.79951

Particular cases:

(i) If the maintenance is so planned and the equipments are so designed that the failure rates of digester and deckers are made negligibly small, i.e. $\alpha \rightarrow 0, \sigma \rightarrow 0$ which may be obtained by performing scheduled maintenance etc., then the steady state availability of the system is given by

$$[A_1] = \left[1 + \frac{\lambda}{\mu} + \frac{\theta}{\phi} + \frac{\lambda\theta}{\mu\phi}\right]\left[\left(1 + \frac{\lambda}{\mu} + \frac{\theta}{\phi}\right)\left(1 + \frac{\lambda\theta}{\mu\phi}\right) + \frac{\lambda^2}{\mu^2} + \frac{\theta^2}{\phi^2}\right]^{-1}$$

(ii) If by proper arrangement of skilled worker and maintenance planning on B and E, we reduce the failure rates to the minimum possible, i.e., $\lambda \rightarrow 0, \theta \rightarrow 0$ which we can do through preventive maintenance etc., then the steady state availability of the system $[A_2] \rightarrow 1$.

Taking some relevant values of $\alpha, \sigma, \lambda, \theta, \beta, \varepsilon, \mu$ and ϕ, the Tables 1 to 5 and Figs 2 to 6 are given as follows:

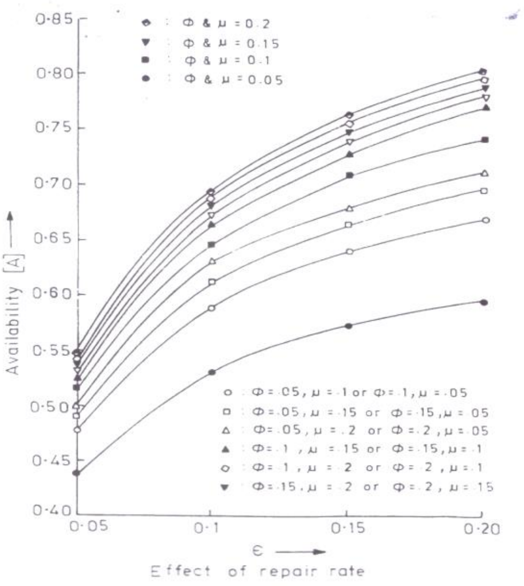

Fig. 6. The effect of repair rate with digester repair rate (*B*) = 0.1.

Table 5: Taking $\beta = 0.1$

ϕ	μ	Availability (*A*)			
		$\varepsilon = 0.05$	$\varepsilon = 0.10$	$\varepsilon = 0.15$	$\varepsilon = 0.20$
0.05	0.05	0.43848	0.53175	0.57233	0.59503
0.05	0.10	0.47771	0.59055	0.64103	0.66964
0.05	0.15	0.49173	0.61212	0.66652	0.69752
0.05	0.20	0.49882	0.62316	0.67962	0.71188
0.10	0.05	0.47771	0.59055	0.64103	0.66964
0.10	0.10	0.51458	0.64795	0.70922	0.74442
0.10	0.15	0.52586	0.66593	0.73082	0.76825
0.10	0.20	0.53097	0.67416	0.74074	0.77922
0.15	0.05	0.49173	0.61212	0.66652	0.69752
0.15	0.10	0.52586	0.66593	0.73082	0.76825
0.15	0.15	0.53549	0.68147	0.74957	0.78899
0.15	0.20	0.53964	0.68819	0.75772	0.79802
0.20	0.05	0.49882	0.62316	0.67962	0.71188
0.20	0.10	0.53097	0.67416	0.74074	0.77922
0.20	0.15	0.53964	0.68819	.75772	0.79802
0.20	0.20	0.54325	0.69407	0.76486	0.80595

To make the system maximum available under existing facilities, the failure rate of each unit is minimised by adopting some measures, viz.,

(i) reducing the delay in information to the section manager,

(ii) providing the instruction manuals and support software,

(iii) reducing the delay in starting the repair,

(iv) preparing skilled workers for the equipment,

(v) making the required tools easily available,

(vi) studying the causes of increased failure rate of units,

(vii) planning the maintenance in advance for various units.

5. DISCUSSION

For the values of $\alpha, \sigma, \lambda, \theta, \beta, \varepsilon, \mu$ and ϕ, the tables and figures exhibit the effect of various parameters upon the system. Tables 1 and 2 and the graphs in Figs 2 and 3 show that if the value of α is doubled or thereabouts, the value of availability changes slowly. Similarly changes in λ and θ do not affect the availability very much (having a standby unit). Generally the failure rate of a digester (α) is very small (failure in any mounting of the digester is repaired/replaced in a small time) and moreover an unskilled or less skilled person may look after the equipment. Furthermore, the cooking process for one trip takes about 8-10 h which is sufficient for 4-6 h for the plant to consume the cooked pulp (depending upon the capacity of the plant). It is also obvious that the failure of the decker has more effect upon availability that the failure of the knotter. The tables show that the failure in the opener has more effect upon availability than the failure in the knotter. The failure rate of the digester may be minimised using scheduled maintenance for the subsystem. Skilled workers are to be provided for the knotter, decker and opener subsystems. These subsystems require more care. Among these, the decker has more effect upon the availability than the others, so it requires more carefulness. The failure rate of the subsystem having the decker may be minimised by performing scheduled maintenance upon the subsystem. Standby units of the knotter and the opener increases the delay in failure of the subsystems (*B* and *E*). Failure rates of these subsystems (*B* and *E*) may be minimised by performing preventive maintenance. Minimisation of α and σ gives higher values of availability as shown in Table 3 and the graph in Fig. 4. The values of availability may be approached up to the neighbourhood of 1.

Tables 4 and 5 and the graphs in Figs 5 and 6 show that a change in repair rates changes the availability considerably. So making of skilled workers, an increase in repair rate, preferably of the knotter, decker and opener, giving higher availability may be achieved. Therefore the management should be more careful about the maintenance planning of the subsystems preferably in the order *D* (decker), *E* (opener), *B* (knotter) and *A* (digester), respectively.

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